



PROOF THAT

$$q_{\max} = \frac{S_f D_j}{4}$$

FOR GREENSHIELD'S EQUATIONS

$$q = S_f \left( D - \frac{D^2}{D_j} \right) \implies q = S_f D - \frac{S_f D^2}{D_j} \quad \text{WHERE } S_f \text{ \& } D_j \text{ ARE CONSTANTS}$$

$$\text{AT } q_{\max} : \frac{dq}{dD} = 0 \implies \frac{dq}{dD} = S_f - \frac{2S_f D}{D_j} = 0 \implies S_f = \frac{2S_f D}{D_j} \implies 1 = \frac{2D}{D_j} \implies D_j = 2D$$

$$\text{AT } q_{\max} : D = \frac{D_j}{2} \implies q_{\max} = S_f \left( \frac{D_j}{2} - \frac{\left(\frac{D_j}{2}\right)^2}{D_j} \right)$$

$$q_{\max} = S_f \left( \frac{D_j}{2} - \frac{D_j}{4} \right) \implies q_{\max} = S_f \left( \frac{D_j}{4} \right) = \underline{\underline{\frac{S_f D_j}{4}}}$$